Possible relation between galactic flat rotational curves and the Pioneers' anomalous acceleration

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Abstract

We consider a generic minimal modification of the Newtonian potential, that is a modification that introduces only one additional dimensional parameter. The modified potential depends on a function whose behavior for large and small distances can be fixed in order to obtain (i) galactic flat rotational curves and (ii) a universal constant acceleration independent of the masses of the interacting bodies (Pioneer anomaly). Then using a dimensional argument we show that the Tully-Fisher relation for the maximal rotational velocity of spiral galaxies follows without any further assumptions. This result suggests that the Pioneer anomalous acceleration and the flat rotational curves of galaxies could have a common origin in a modified gravitational theory. The relation of these results with the Modified Newtonian Dynamics (MOND) is discussed.

Key words: Pioneer anomaly, MOND, Tully-Fisher, flat rotational curves, anomalous acceleration
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1 Introduction

In 1998 Anderson et al. [1] reported an unmodeled constant acceleration towards the Sun of about $a_P \simeq 8.5 \times 10^{-8}\mathrm{cm/s}^2$ for the spacecrafts Pioneer 10 (launched 2 March 1972), Pioneer 11 (launched 4 December 1973), Galileo (launched 18 October 1989) and Ulysses (launched 6 October 1990).

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In a subsequent report [2] they discussed in detail many suggested explanations for the effect and gave the value $a_P = (8.74 \pm 1.33) \times 10^{-8}\text{cm/s}^2$ directed towards the Sun.

The data covered many years staring in 1980 when due to the large distance ($\simeq 20$ AU) of Pioneer 10 from the Sun the solar radiation pressure became sufficiently small to look for unmodeled accelerations. The data was collected up to 1990 for Pioneer 11 ($\simeq 30$ AU) and up to 1998 ($\simeq 70$ AU) for Pioneer 10.

The spacecraft masses were quite different. Galileo had a mass $m_G = 1298$ kg (the orbiter mass at launch minus the propellent) while the Pioneers had a mass $m_P = 223$ kg. Their orbits were also quite different, nevertheless the same constant centripetal acceleration was detected. This acceleration (the Pioneer anomaly) does not appear in the planet ephemerides or in the accurate range measurement of the Viking mission [2]. It has been later confirmed that no such anomalous centripetal acceleration affects the motion of the major planets of the solar system [6,13]. If confirmed the effect would imply a violation of the equivalence principle as heavy bodies with the mass of a planet would fall differently from lighter bodies like the spacecrafts. However, the different masses of the Galileo and Pioneer spacecrafts show that the anomalous acceleration is independent of the mass of the free falling bodies as long as they have a small mass, a fact that does not help in clarifying why the planets are not subject to the anomalous acceleration. A systematic error would clearly solve this problem but so far none has been found. In has been suggested [10] that within some years the observation of the motion of minor trans-Neptunian objects could finally confirm or rule out, by filling the observational mass gap, the presence of an anomalous acceleration.

In this work we shall consider the Pioneer anomalous acceleration as real and, in order to avoid the difficulties for the different behavior of light and heavy bodies, we shall consider 'test particles' of mass m' in the field of an heavier body m, $m' \ll m$. The concept of test particle we use is a relative concept related to the ratio m'/m, thus, for instance, the Pioneer spacecrafts are test particles in the field of the Sun ($\log \frac{m}{m'} \simeq 28$), the stars at the outer edge of a galaxy are test particles in the field of the galaxy ($\log \frac{m}{m'} \simeq 10$) and the major planets of the Solar system are not test particles ($\log \frac{m}{m'} \lesssim 8$). For a more precise definition, that would clarify the origin of the assumed breaking of the equivalence principle, one would need a more complete physical theory than that provided in this work.

Soon afterwards Anderson's report it was noted by many authors that there is a numerical coincidence between $a_P/c = (2.8 \pm 0.4) \times 10^{-18}\text{s}^{-1}$ and the Hubble constant [5],

$$H_0 = (72 \pm 8)\text{km/(s Mpc)} = (2.3 \pm 0.3) \times 10^{-18}\text{s}^{-1}.$$

This observation suggests that the Pioneer acceleration could be a short range (at the size of the solar system) universal acceleration, that is a constant acceleration unrelated with the masses of the test particles and the Sun. In this work we shall make this hypothesis

(i) Test particles in the field of an heavy body of mass m have at small distances an acceleration $\mathbf{a} = a(r)\hat{\mathbf{e}}_r$ with $a(r) = -G\frac{m}{r^2} - a_P$, where a_P is a universal constant that does not depend on m, and r is the distance between the test particle and the mass m.

Note that the physics underlying this assumption must be complemented by defining the effect of the lighter body on the heavier one. The compatibility with the conservation of momentum implies that the heavier body is attracted by the lighter one with an acceleration $|a(r)| = G\frac{m'}{r^2} + \frac{m'}{m}a_P$. Since a_p is small and m'/m is small the anomalous acceleration on the heavier body would be undetectable.

Our second assumption is based on the observation that the rotational curves of spiral galaxies are asymptotically flat. The velocity of the stars and the hydrogen atoms far from the galactic center does not fall as $v \sim \sqrt{Gm/r}$ as in Newtonian gravity, instead it tends to a constant v_∞. This fact is usually explained with the presence of an invisible dark matter halo around the galaxies which modifies the effective gravitational potential. Let us denote with $\mathbf{a} = a(r)\hat{\mathbf{e}}_r$ the acceleration field generated by a mass m. Our second assumption is

(ii) $\lim_{r\to+\infty} ra(r) = -v_\infty^2 = cnst.$

so that flat rotational curves follow.

2 The modified potential

The conditions (i) and (ii) are not satisfied by a Newtonian potential, therefore we look for a minimal modification of the Newtonian potential. By minimal modification we mean a modification that introduces only one auxiliary dimensional parameter. Without loss of generality the modified potential can be written in the form

$$V(r) = -G\frac{m}{r}(1 + f(\beta r)), \tag{1}$$

where $\beta \in [L]^{-1}$ is the new dimensional parameter and $f(x)$, $x = \beta r$, is a C^2 function $f : \mathbb{R} \to \mathbb{R}$.

We are going to find some constraints on the function f so as to satisfy (i) and (ii). First, the Newtonian limit implies $f(0) = 0$. Note that since V is defined only up to a constant, the function $f(r)$ is defined only up to linear terms in r. Note also that there is a rescaling freedom in the definition of f and β, indeed let $\lambda \in \mathbb{R} - \{0\}$ and redefine

$$\bar{f}(x) = f(\lambda x), \tag{2}$$
$$\bar{\beta} = \beta/\lambda, \tag{3}$$

then $\bar{f}(\bar{\beta}r) = f(\beta r)$. Under the assumption $|f''(0)| \neq 0$, we use this freedom to fix $|f''(0)| = 2$ and $\beta > 0$.

The acceleration field $\mathbf{a} = a(r)\hat{\mathbf{e}}_r = -\nabla V$ is given by (the derivatives are with respect to x)

$$a(r) = -G\frac{m}{r^2}(1+f) + G\frac{m\beta}{r}f'. \tag{4}$$

Let us consider the condition (i). Taylor expanding $f(x)$ and $f'(x)$ at $x = 0$ we obtain the acceleration field at small distances

$$a(r) = -\frac{Gm}{r^2} + \frac{Gm}{2}\beta^2 f''(0) + Gm\beta^2 O(\beta r). \tag{5}$$

The condition (i) is satisfied iff $f''(0) < 0$, which due to our normalization implies $f''(0) = -2$, and

$$\beta^2 = a_P/Gm \tag{6}$$

where a_P is a universal constant independent of m. Consider the spacecraft Pioneer in the solar system. The last term on the right-hand side of Eq. (5), is much smaller than the second one since $a_P/(Gm_\odot/d^2) < 10^{-3}$, where $d < 87AU$ is the Pioneer distance from the Sun and $m_\odot$ is the mass of the Sun.

The condition (ii) implies

$$\lim_{r\to+\infty} \frac{ra(r)}{Gm\beta} = \lim_{x\to+\infty}(f' - f/x) = -\frac{v_\infty^2}{Gm\beta}. \tag{7}$$

Note that x is a dimensionless parameter, it follows that as $r \to \infty$, $f(\beta r) \to f_\infty(\beta r)$ a function that solves the differential equation

$$f'_\infty - f_\infty/x = -K_1, \qquad K_1 \in \mathbb{R}^+, \tag{8}$$

and $v_\infty^2 = K_1 G m \beta$. Using the relation between β and a_P we obtain the Tully-Fisher relation

$$v_\infty^4 = (K_1^2 a_P G) m, \tag{9}$$

which expresses the proportionality between the mass (and hence the luminosity) of the spiral galaxy and the fourth power of the asymptotic rotational velocity.

We recall that, more generally, the Tully-Fisher relation states $L \propto v_\infty^p$ where L is the luminosity of the galaxy. Observationally the wave-band dependent exponent p stays in the range $[2.5, 5]$, and has the smallest scatter in the near infrared for which p is found to be close to 4 (see [7]).

The solution of Eq. (8) is

$$f_\infty = K_2 \beta r - K_1 \beta r \ln(\beta r), \tag{10}$$

where K_2 is an integration constant. As observed previously we may redefine the potential so that $K_2 = 0$. Alternatively the linear freedom of f can be fixed requiring $f'(0) = 0$. We make the latter choice. The constant K_1 is expected to be of the order of unity since it is the finite limit of a dimensionless function $f(x)/(-x \ln x)$ that comes from a yet unknown gravitational theory. As a consequence the proportionality constant in the Tully-Fisher relation is related to the Pioneer acceleration and therefore, according to our model, should be of the order of $H_0 G c$. It has long be recognized that the proportionality constant in the Tully-Fisher relation has exactly the predicted magnitude [9].

3 The relation with MOND

It is interesting to explore how the modified potential dynamics is related to the MOND theory [8,9,11]. Let us introduce the Newtonian acceleration $g_N = Gm/r^2$, the MOND characteristic acceleration $a_0 = K_1^2 a_P$ and the function $z(y)$, (with $y = 1/(K_1 x)^2 = g_N/a_0$)

$$z(y) = y[1 + f(\frac{1}{K_1\sqrt{y}})] - \frac{\sqrt{y}}{K_1} f'(\frac{1}{K_1\sqrt{y}}), \tag{11}$$

then Eq. (4) can be rewritten

$$a/a_0 = -z(g_N/a_0). \tag{12}$$

For problems with spherical or cylindrical mass configurations the theory reduces to MOND provided $z(y)$ has an inverse $I(z) = z\mu(z)$, $I(z(y)) = y$, with $\mu(z) \sim z$ for $z \ll 1$ and $\mu(z) \sim 1$ for $z \gg 1$.

Let us see whether these conditions are compatible with the modified potential. The limit $x \to +\infty$, corresponds to $y \to 0$ and the asymptotic behavior of $f(x)$ implies that $z \to 0$ as $z(y) \sim \sqrt{y}$ and hence $\mu(z) \sim z$. The limit $x \to 0$ corresponds to $y \to +\infty$ and $f(0) = 0$ implies that $z \to +\infty$ as $z(y) \sim y$ and hence $\mu(z) \to 1$. Thus for symmetric mass configurations we recover MOND.

Some comments are in order. The function $z(y)$ does not necessarily need to be invertible for certain choices of $f(x)$, hence a 'modified inertia' formulation in terms of a function μ is not guaranteed. The asymptotic behavior of $f(x)$ for $x \to 0, +\infty$ used above does not suffice to recover MOND from the minimally modified potential. Indeed, we used also condition (i) that led to a constraint for β and then to the functional form (11) for $z(y)$. In our modified potential formulation the Tully-Fisher relation is derived from the unrelated assumption (i) while in MOND it follows by construction from the condition $\mu(z) \sim z$ for $z \ll 1$. MOND does not give necessarily the Pioneer anomaly that instead is accommodated since the beginning in our formulation. In MOND the Pioneer anomaly would be included imposing the additional constraint $f''(0) = -2$. It is not difficult to show that it corresponds to

$$\mu(z) \sim 1 - \frac{1}{K_1^2 z}, \quad \text{as} \quad z \to +\infty. \tag{13}$$

Since the differences between MOND and our derived dynamics are only minimal we can regard these calculations as a proof that a MOND theory subject to constraint (13) follows from assumptions (i) and (ii). Note that MOND does not satisfy the equivalence principle a fact which is coherent with the phenomenology related to the Pioneer anomaly from which we started.

The observations give (through Eq. (9)) $a_0 = 1.2 \times 10^{-8}\text{cm/s}^2$, and we find from the relation $a_0 = K_1^2 a_P$ that $K_1^{-2} \simeq 7$. We stress again that since K_1 is of the order of unity the dimensional argument is sound.

Some work has been done to restrict the function $\mu(z)$ of MOND theory. In particular we may ask whether the constraint (13) is compatible with the observations. Sereno and Jetzer [12] pointed out that a similar asymptotic behavior is incompatible with the accurate data available on Mars orbit. However, this is not a surprise, since in order to take the Pioneer anomaly seriously we assumed since the beginning that the equivalence principle has to be violated. Clearly the chosen μ function, or equivalently the f function, can not fit the Newtonian orbits of the major planets since it has been chosen in order to reproduce the Pioneer anomaly which, as we said in the introduction, does not

affect the orbit of those planets. We had to make a choice: whether to fit the f function to reproduce a typical Newtonian behavior or to fit it with the data on the Pioneer anomaly. We chose the latter possibility so as to be in a small test particle limit. The choice to consider the problem in some limit makes sense taking into account that the theory was already expected to violate the equivalence principle.

Apart from the constraint (13) the function μ of our theory is a typical μ function of MOND theory. Famaey and Binney [4,14] showed that the simple choice $\tilde{\mu}(z) = z/(1+z)$ is particularly successful in fitting the Milky Way and the galaxy NGC3198, in particular, it proved superior than the traditional choice $\check{\mu} = z/\sqrt{1+z^2}$. It is interesting to note that the function $\tilde{\mu}$ has the asymptotic behavior $\tilde{\mu} \sim 1 - \frac{1}{z}$, and hence, although it is not compatible with our theory (as the expansion is slightly different from the one required) it implies the presence of an anomalous centripetal acceleration of value a_0. Nevertheless, a fit of the Milky Way led Famaey and Binney [4] to conclude that the function $\tilde{\mu}$ gives good results only up to values $z \lesssim 5$, while for the Pioneers we are in the range $z \sim 10^3$. At that range Famaey and Binney argue that a transition should have already taken place to the function $\check{\mu}$, which unfortunately does not imply a universal centripetal acceleration. Although, these results are not conclusive, as they are based on the study of only two galaxies, they seem to imply that the Pioneer anomalous acceleration does not show up in the dynamics of galaxies.

We end the section by noticing that since the Pioneer anomaly is mainly a post-Newtonian effect it was natural to start our study from Galilei invariant assumptions (i) and (ii). Nevertheless, the closeness of the final theory to MOND and the possibility of generalizing MOND in a relativistic way [3] clarifies that those assumptions were not incompatible with relativistic physics as long as they are taken in the suitable slow-speed weak-gravity limit.

4 Conclusions

The galactic flat rotational curves and the Pioneer anomaly are among the few phenomena that could suggest a departure from the Newtonian gravitational potential. In this work we assumed a common origin in a modified but yet unknown (effective) gravitational theory. We considered the case of two masses, $m' \ll m$, and introduced a minimally modified potential, that is a potential that involves only one auxiliary dimensional parameter. We showed that such potential can be suitably tuned to produce the said phenomena. We found that a minimally modified potential $V(r)$ that meets conditions (i) and

(ii) has the form

$$V(r) = -G\frac{m}{r}[1 + f(\sqrt{\frac{a_P}{Gm}}\ r)], \tag{14}$$

where function $f(x)$ satisfies $f(0) = 0$, $f'(0) = 0$, $f''(0) = -2$, and has the asymptotic behavior $f \sim -K_1 x \ln x$. Functions of this kind exist, consider for instance the simple choice, $f(x) = -K_1 x \ln(1 + x/K_1)$. Moreover, the asymptotic rotational velocity is related to K_1 by $K_1 = \frac{v_\infty^2}{\sqrt{a_P Gm}}$, and hence the Tully-Fisher relation holds. The emergence of the Tully-Fisher relation can be considered as a suggestion that the starting assumption, i.e. that the flat rotational curves and the Pioneer anomaly have the same gravitational origin, could indeed be correct. Nevertheless, we showed that under the said assumptions we recover a MOND like theory subject to the constraint (9). Most of the successful predictions of MOND theory do not depend on the particular form of the $\mu(z)$ function (i.e. on $f(x)$), however, in the last years some work has been done to constraint the function μ by using galactic rotation curves. Using this data and results by other authors we concluded that it is unlikely that the Pioneer anomalous acceleration shows up at the galactic scales. In other words although the assumption of the Pioneer anomaly naturally leads to the the successful MOND theory, the latest information available on galaxies seems to exclude the presence of an anomalous centripetal acceleration of the order of a_P. This complex situation seems to require more investigation, the possibility of a relation between galactic dynamics and the Pioneer anomaly being still open.

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